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Higher derivative operators from transmission of supersymmetry breaking on S_1/Z_2

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Abstract: We discuss the role that higher derivative operators play in field theory orbifold compactifications on S_1/Z_2 with local and non-local Scherk-Schwarz breaking of supersymmetry. Integrating out the bulk fields generates brane localised higher derivative counterterms to the mass of the brane (or zero-mode of the bulk) scalar field, identified with the Higgs field in many realistic models. Both Yukawa and gauge interactions are considered and the one-loop results found can be used to study the "running" of the scalar field mass with respect to the momentum scale in 5D orbifolds. In particular this allows the study of the behaviour of the mass under UV scaling of the momentum. The relation between supersymmetry breaking and the presence of higher derivative counterterms to the mass of the scalar field is investigated. This shows that, regardless of the breaking mechanism, (initial) supersymmetry cannot, in general, prevent the emergence of such operators. Some implications for phenomenology of the higher derivative operators are also presented.

Keywords: Field Theories in Higher Dimensions, Large Extra Dimensions, Supersymmetry Breaking, Supersymmetric Effective Theories.

Contents

1. Introduction

The physics of extra dimensions is receiving a strong research interest in the context of effective field theory approaches to compactification, sometimes referred to as "field theory orbifolds". While string theory may provide the ultimate description of compactification, field theory orbifolds can also consistently (re)address interesting issues such as supersymmetry breaking, the hierarchy problem, radiative corrections or their experimental signatures. In this work we investigate the implications of supersymmetry breaking in 5D field theory orbifolds for the one-loop corrections to the mass of the 4D scalar fields.

An aspect that is somewhat overlooked in recent studies of radiative corrections in gauge theories on field theory orbifolds (see for example [1]–[9]), is the role of higher derivative operators.[1] In general it is thought that such operators, being higher dimensional, are suppressed if the scale where they become relevant is high enough. However, from a 4D point of view the most natural such scale is the compactification scale $1/R$ and,

[1] For works on this topic see for example [10]–[13].

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if this is low (TeV scale), such operators can have a significant effect even at low energies. For generality we shall consider the role of such operators in orbifolds with a radius R of an arbitrary value (large or small).

The models that we investigate are 5D N=1 supersymmetric, compactified on S_1/Z_2 and the interactions are localised superpotential (in the extra dimension) and gauge interactions. These are standard interactions in any higher dimensional theory which aims to recover after compactification and at low energies, the Standard Model (SM) or its supersymmetric versions. Such interactions induce loop corrections to the mass of the zero mode scalar field (if this is a bulk field) or to that of a brane scalar field. It is important to note that this field is regarded in many models as the SM Higgs field [1]–[8]. Therefore such loop corrections are important for the hierarchy problem, electroweak symmetry breaking or running of the mass of the scalar field.

Recent studies of loop corrections to the mass of the 4D scalar field showed that radiative corrections are of type (see for example [1, 3])

$$m_{\phi_H}^2 \sim \eta \frac{\alpha^2}{R^2}, \tag{1.1}$$

where $\eta = -1\,(+1)$ when α is the Yukawa (gauge) coupling. As a result one can have radiative electroweak symmetry breaking[2] triggered by towers of Kaluza-Klein states of bulk fields which couple, via the aforementioned interactions, to the (zero-mode or brane) scalar field associated with the Higgs field. Note that in the absence of a rigorous mechanism to fix $1/R$ to low values (TeV or so) such results do not provide a solution to the hierarchy problem even at one-loop. But they open new ways to address these problems and we think this is worth further investigation.

As we shall see shortly, it turns out that the non-renormalisable character of these theories becomes apparent earlier than one may like, but with potentially interesting consequences. That means that results like (1.1) are in general altered by the effects of higher derivative operators, in some cases already at one loop. Their effect depends on the size of the radius R, but as already mentioned, here we keep R arbitrary. The result of our one-loop Yukawa corrections shows that

$$m_{\phi_H}^2(q^2) = m_{\phi_H}^2(0) + \xi\, q^4\, R^2 + \frac{1}{R^2}\,\mathcal{O}(q^2 R^2), \qquad m_{\phi_H}^2(0) \sim -\frac{\alpha^2}{R^2}, \tag{1.2}$$

where ξ is an (unknown) coefficient of the higher derivative operator.

Eq. (1.2) shows that the scalar mass depends on both R and $1/R$ which[3] we find rather interesting. Further, if R is somehow fixed to a large value (inverse TeV scale) to explain a low mass for the scalar without large fine tuning, the second term in the first equation becomes more important. Given the unknown coefficient ξ of the higher derivative operators, this can affect the predictive power of the models. Further difficulties at consistency level may arise for special values of this parameter, as we discuss in section 4. Conversely, if R is

[2] assuming a vanishing mass at the tree level.

[3] Such dependence of loop corrections on both R and $1/R$ is familiar in one-loop string corrections (T duality) such as those to gauge couplings [14].

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very small (q^2 fixed), the role of higher derivative operators is suppressed, but then the first term re-introduces the quadratic mass scale (hierarchy) problem at one-loop, familiar from the Standard Model. While this is the general picture, a detail analysis should consider the $\mathcal{O}(q^2R^2)$ terms in (1.2) whose expression can be computed in general cases using our technical results. Finally, our calculation can be used to re-address previous studies of the radiative electroweak symmetry breaking induced by towers of Kaluza-Klein modes, and to investigate the one-loop running of the scalar mass and its ultraviolet behaviour under the UV scaling of the momenta, $q^2 \to \sigma q^2$.

We investigate whether the presence of higher derivative operators and the above considerations depend on the way 5D N=1 supersymmetry is broken. This is the main purpose of the paper. We consider both local and non-local supersymmetry breaking, transmitted to the visible sector via radiative corrections. The non-local breaking includes discrete and continuous Scherk-Schwarz twisted boundary conditions for the bulk fields. The plan of the paper is as follows: in section 2 we discuss the orbifold compactifications and supersymmetry breaking by local F terms of a brane field at a hidden brane (section 2.1) and non-local breaking (section 2.2) by Scherk-Schwarz boundary conditions. We show that higher derivative operators are generated as counterterms for Yukawa but not for gauge interactions. It turns out that the presence of these counterterms is rather independent of the above ways of supersymmetry breaking and this is discussed in section 3. Some phenomenological implications of the higher derivative operators that are found are discussed in section 4. Appendix A.1 contains formulae relevant for the study of the running of the scalar field mass (with the momentum scale), which may also be used in other applications. Appendix A.2 outlines a dimensional analysis needed in section 4.

2. Higher derivative counterterms on S_1/Z_2

On the S_1/Z_2 orbifold, one can consider vector supermultiplets and hypermultiplets. The former may be described in a 4D language as made of vector superfield $V(\lambda_1, A_\mu)$ and an adjoint chiral superfield $\Sigma((\sigma+iA_5)/\sqrt{2}, \lambda_2)$, where $\lambda_{1,2}$ are Weyl fermions, σ a real scalar and A_μ, A_5 the 5D gauge field. The hypermultiplet contains two chiral superfields $\Phi(\phi,\psi)$ and $\Phi^c(\phi^c,\psi^c)$ with opposite quantum numbers, and where ϕ, ϕ^c are complex scalars and ψ, ψ^c are Weyl fermions. The orbifold conditions considered are such as the gauge field A_μ has even parity (has a massless zero mode) to respect the 4D gauge invariance. We consider the following parity assignments

$$\begin{aligned} \Phi(x,-y) &= \Phi(x,y)\,, & V(x,-y) &= V(x,y) \\ \Phi^c(x,-y) &= -\Phi^c(x,y) & \Sigma(x,-y) &= -\Sigma(x,y)\,, \end{aligned} \tag{2.1}$$

where Φ can be any of the SM fields Q, U, D, L, E. As a result, the original 5D N=1 supersymmetry is broken and the fixed points ($y = 0, \pi R$) of the orbifold have a remaining 4D N=1 supersymmetry. Further, we consider the following localised interaction,

$$\mathcal{L}_4 = \int dy \delta(y) \left\{ -\int d^2\theta \left[\lambda_t QUH_u + \lambda_b QDH_d + \cdots\right] + \text{h.c.} \right\}. \tag{2.2}$$

The 5D coupling $\lambda_t = f_{5,t}/M_*^n = (2\pi R)^n f_{4,t}$ where $f_{5,t}$ ($f_{4,t}$) is the dimensionless 5D (4D), M_* is the cutoff of the theory. In the following Q, U, D superfields are *always* bulk fields and have mass dimension $[Q]=[U]=[D]=3/2$, while the Higgs field $H_{u,d}$ can be a brane field $[H_{u,d}] = 1$ if $n = 1$ or a bulk field[4] $[H_{u,d}] = 3/2$, when $n = 3/2$ (when it also has a $H^c_{u,d}$ partner). The above spectrum and interactions define our minimal model. While this is not phenomenologically viable, it is sufficient to illustrate the idea of the paper. Moreover the spectrum and interaction considered are generic in many detailed models which reproduce the SM or its supersymmetric versions. Such models, for which our findings are relevant, can be found in refs. [1] to [8].

The purpose of this work is to investigate the correction to the mass of the brane (or zero-mode of the bulk) scalar fields ϕ_{H_u}, ϕ_{H_d} induced[5] by towers of Kaluza-Klein states of Q, U, D superfields, via interaction (2.2). Gauge corrections will also be considered. Since similar considerations apply to both H_u and H_d, we shall compute the one-loop correction to the mass m_{ϕ_H} of ϕ_{H_u}, hereafter denoted simply ϕ_H. To compute the one-loop corrected m_{ϕ_H} one must first evaluate the spectrum of Kaluza-Klein modes, using eq. (2.1). However this spectrum also depends on the mechanism of the further breaking of 4D N=1 supersymmetry which is then transmitted to the "visible" sector at $y = 0$. We shall distinguish two cases discussed separately: **I.** Localised supersymmetry breaking and **II.** Non-local supersymmetry breaking.

2.1 Higher derivative operators from localised supersymmetry breaking

We consider that supersymmetry is broken at a distant (hidden) brane located at $y = \pi R$ by

$$\mathcal{L}_4 = \int dy \delta(y - \pi R) \left[\int d^2\theta M_*^2 \, Z + h.c. \right], \tag{2.3}$$

where Z is a (gauge singlet) brane field at $y = \pi R$. The bulk fields Q, U feel the supersymmetry breaking via the couplings

$$\mathcal{L}_4 = \int dy \delta(y - \pi R) \left\{ - \int d^4\theta \left[\frac{c_Q}{M_*^3} \, Q^\dagger Q \, Z^\dagger Z + \frac{c_U}{M_*^3} \, U^\dagger U \, Z^\dagger Z \right] \right\}. \tag{2.4}$$

Therefore, when $\langle Z \rangle \sim F_Z \theta^2$, the bulk fields such as $\phi_{Q,U}$ with non-zero coupling at the $y = \pi R$ brane have the spectrum modified. One can show that (for details see for example [3])

$$\tan[\pi R m_{\phi_M,n}] = \frac{c_M}{2} \frac{F_Z^2}{M_*^4} \frac{M_*}{m_{\phi_M,n}} \, , \Rightarrow m_{\phi_M,n} = \left(n + \frac{1}{2}\right) \frac{u}{R} \, , \qquad M = Q, U \, . \tag{2.5}$$

With the choice $c_M \sim \mathcal{O}(1)$, $F_Z \sim M_*^2$, and to leading order in $1/(RM_*)$, one can set $u = 1$. This is usually referred to as "strong" supersymmetry breaking, otherwise u is a series in $1/(RM_*)$.

[4] If $H_{u,d}$ are bulk fields they also satisfy a condition similar to that for Φ in eq. (2.1).

[5] One needs two Higgs fields of opposite hypercharge to avoid (quadratically divergent) FI corrections [15].

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Unlike the fields $\phi_{Q,U}$, their fermionic partners $\psi_{Q,U}$ do not couple to the vev of Z, and their mass is not changed by (2.3). This results in the breaking at $y = \pi R$ of the remaining 4D N=1 supersymmetry of zero modes. Similarly, the fields $\psi^c_{Q,U}$, $\phi^c_{Q,U}$ do not couple to Z (see eq. (2.1)), thus their mass spectrum is not affected. Finally, the Higgs field H_u (also H_d) is considered to be a *brane* field, localised at $y = 0$, so it does not have tree level interactions with the field Z. However, it feels the supersymmetry breaking at $y = \pi R$ through loops of the bulk fields Q, U, (or Q, D for H_d) via eq. (2.2). Therefore the spectrum of the bulk fields is, using (2.1), (2.5)

$$\begin{aligned} m_{\psi_M,k} &= \frac{k}{R}, \qquad k \geq 0, \qquad m_{\psi^c_M,k} = \frac{k}{R}, \quad k \geq 1 \\ m_{\phi_M,k} &= u\left(\frac{k + r^\phi}{R}\right), \quad k \geq 0, \qquad m_{\phi^c_M,k} = \frac{k}{R}, \quad k \geq 1, \qquad M \equiv Q, U. \end{aligned} \tag{2.6}$$

where $r^\phi = 1/2$ and $u = 1$ for strong supersymmetry breaking. The case with $u \neq 1$ is discussed in section 3. Finally, the wavefunction normalisation coefficients (at $y = 0$) are $\eta_k^{\phi_M} = 1$, $\eta_k^{\psi_M} = \eta_k^{F_M} = 1/\sqrt{2}^{\delta_{k,0}}$ ($k \geq 0$). These results will be used in section 2.1.1 to compute m_{ϕ_H} at one-loop.

Finally, let us also consider the case of supersymmetry breaking at $y = \pi R$ with a brane-localised gaugino mass on S^1/Z_2,

$$\mathcal{L}_4 = \int dy \delta(y - \pi R) \int d^2\theta \frac{1}{(4g_5)^2} \frac{c_W}{M_*^2} Z \text{Tr}\Big[W^\alpha W_\alpha\Big] + \text{h.c.} \tag{2.7}$$

Here W_α contains gaugino as its lowest component. Note that only the even parity gaugino λ_1 (see eq. (2.1)) couples to Z and feels the supersymmetry breaking at $y = \pi R$. Further, λ_1 and λ_2 are coupled through their kinetic term. Eq. (2.7) is the counterpart of eq. (2.4) for the gauge case. Using the equation of motion one finds [3]

$$\tan[\pi R m_\lambda] = \frac{c_W}{4M_*^2} F_Z, \tag{2.8}$$

Then, the spectrum of gaugino is

$$m_{\lambda_1} = m_{\lambda_2} = \frac{k + \rho}{R}, \qquad k \in \mathbf{Z}, \tag{2.9}$$

with

$$\rho = \frac{1}{\pi} \arctan\left(\frac{c_W F_Z}{4M_*^2}\right). \tag{2.10}$$

This result is used in section 2.1.2 for the one-loop gauge correction to the scalar mass m_{ϕ_H}.

2.1.1 One-loop mass correction due to Yukawa interaction

In the on-shell formulation, the interaction in eq. (2.2) becomes in component fields [3]

$$\mathcal{L}_4 = \sum_{k=1}^{\infty} \sum_{l=0}^{\infty} (2\, f_{4,t}) \Big[m_{\phi^c_Q,k} \eta_k^{F_Q} \eta_l^{\phi_U}\, \phi^{c\,\dagger}_{Q,k}\, \phi_{U,l} \phi_H + \text{h.c.} + (Q \leftrightarrow U)\Big] -$$

$$
-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}(2\,f_{4,t})^2\left[\eta_k^{\phi_Q}\eta_l^{\phi_Q}(\eta_m^{F_U})^2\phi^\dagger_{Q,k}\phi_{Q,l}\phi^\dagger_H\phi_H+(Q\leftrightarrow U)\right]-
$$

$$
-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(2\,f_{4,t})\left[\eta_k^{\psi_Q}\eta_l^{\psi_U}\psi_{Q,k}\psi_{U,l}\phi_H+\text{h.c.}\right]. \tag{2.11}
$$

Note that the sum over k in the first line is from $k\geq 1$ since the field ϕ^c is odd under orbifolding, eq. (2.1). If we work on-shell (off-shell) there are three (two) types of diagrams contributing to the mass of brane Higgs field ϕ_H, see figure 1. The one-loop contributions in euclidean space are[6]

$$
\begin{aligned}
-\,im^2_{\phi_H}(q^2)\Big|_B &= i(2f_{4,t})^2N_c\sum_{k\geq0,l\geq0}\left[\eta_k^{F_Q}\eta_l^{\phi_U}\right]^2\int\frac{d^dp}{(2\pi)^d}\frac{(-1)(p+q)^2\mu^{4-d}}{((p+q)^2+m^2_{\phi^c_Q,k})(p^2+m^2_{\phi_U,l})}+\\
&\quad+(Q\leftrightarrow U) \qquad\qquad (2.12)\\
-im^2_{\phi_H}(q^2)\Big|_F &= i(2f_{4,t})^2N_c\sum_{k\geq0,l\geq0}\left[\eta_k^{\psi_Q}\eta_l^{\psi_U}\right]^2\int\frac{d^dp}{(2\pi)^d}\frac{2p.(p+q)\mu^{4-d}}{((p+q)^2+m^2_{\psi_Q,k})(p^2+m^2_{\psi_U,l})}\,.
\end{aligned}
$$

Here $d=4-\epsilon$ $(\epsilon\to0)$, N_c is the number of colours, μ is the scale introduced by the DR scheme, q^2 is the external momentum and the two double sums extend to infinity. The index $B,(F)$ stands for bosonic (fermionic) contributions. One then uses the spectrum (2.6) and coefficients η given after this equation, perform the integrals over p in DR, then the double sums, to find

$$
\begin{aligned}
-\,m^2_{\phi_H}(q^2)\Big|_B &= (2f_{4,t})^2\,N_c\sum_{k\geq0,l\geq0}\frac{1}{2^{\delta_{k,0}}}\int\frac{d^dp}{(2\pi)^d}\frac{(-2)(p+q)^2\mu^{4-d}}{((p+q)^2+k^2/R^2)(p^2+(l+1/2)^2/R^2)}\\
&= -\frac{(2f_{4,t})^2\kappa_\epsilon}{2(4\pi R)^2}N_c\int_0^1 dx\left\{\frac{2-\epsilon/2}{\pi}\mathcal{J}_2[1/2,0,c]+q^2R^2(1-x)^2\mathcal{J}_1[1/2,0,c]\right\}\\
-m^2_{\phi_H}(q^2)\Big|_F &= (2f_{4,t})^2\,N_c\sum_{k\geq0,\,l\geq0}\frac{1}{2^{\delta_{k,0}}}\frac{1}{2^{\delta_{l,0}}}\int\frac{d^dp}{(2\pi)^d}\frac{2p.(p+q)\mu^{4-d}}{((p+q)^2+k^2/R^2)(p^2+l^2/R^2)}\\
&= \frac{(2f_{4,t})^2\kappa_\epsilon}{2(4\pi R)^2}N_c\int_0^1 dx\left\{\frac{2-\epsilon/2}{\pi}\mathcal{J}_2[0,0,c]+q^2R^2x(x-1)\mathcal{J}_1[0,0,c]\right\}, \qquad (2.13)
\end{aligned}
$$

where $\kappa_\epsilon\equiv(2\pi\mu R)^\epsilon$. The functions $\mathcal{J}_{1,2}$ have the following definition and leading behaviour in ϵ

$$
\begin{aligned}
\mathcal{J}_j[c_1,c_2,c] &\equiv \sum_{k_1,k_2\in\mathbf{Z}}\int_0^\infty\frac{dt}{t^{j-\epsilon/2}}e^{-\pi t(c+a_1(k_1+c_1)^2+a_2(k_2+c_2)^2)}=\frac{(-\pi c)^j}{j\sqrt{a_1a_2}}\left[\frac{2}{\epsilon}\right]+\mathcal{O}(\epsilon^0)\,,\quad j=1,2\,.\\
a_1 &= (1-x)\,,\qquad a_2=x\,,\qquad c=x(1-x)\,q^2R^2\,. \qquad (2.14)
\end{aligned}
$$

[6]This form of the radiative corrections can be shown to be equivalent to that derived using 5D Green functions in mixed positions-momentum space (computed in ref. [3], eqs.(57), (60), (64)), and evaluated at $y=0$. This also suggests the (brane) localisation of the corresponding counterterm, see later.

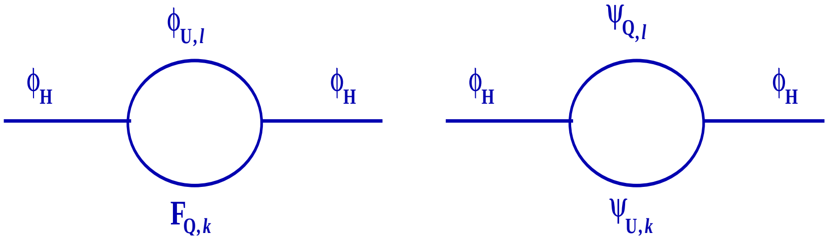

Figure 1: Diagrams contributing to the two-point Green function of the (brane or zero mode of the bulk) scalar field ϕ_H. For the left diagram one should also add the similar contribution with $Q \leftrightarrow U$.

For a complete expression of the functions $\mathcal{J}_{1,2}$ see the appendix, eqs. (A.1) to (A.3) and (A.11) to (A.15). It is important to notice that the leading (divergent) behaviour of $\mathcal{J}_{1,2}$ depends on c and a_1, a_2 but is independent of c_1, c_2. The dependence on c is very important, since it is only for $c \sim q^2 R^2 \neq 0$ i.e. non-zero external momentum that one is able to "see" the poles of $\mathcal{J}_{1,2}$. After adding the bosonic and fermionic contributions in (2.13), one finds

$$\begin{aligned} -\, m^2_{\phi_H}(q^2) = \frac{(2 f_{4,t})^2}{2(4\pi R)^2} N_c \bigg\{ \int_0^1 dx \left(\frac{2}{\pi}\right) \Big[\mathcal{J}_2[0,0,c] - \mathcal{J}_2[1/2,0,c]\Big] + \\ + \kappa_\epsilon(q^2R^2) \int_0^1 dx \Big[x(x-1)\mathcal{J}_1[0,0,c] - \\ - (1-x)^2\, \mathcal{J}_1[1/2,0,c]\Big]\bigg\} \end{aligned} \tag{2.15}$$

with c as in (2.14). Note that if $q^2 = 0$ the second line above is absent, so $m^2_{\phi_H}(q^2=0)$ is given by the first line alone. Further, in the difference $\mathcal{J}_2[0,0,c] - \mathcal{J}_2[1/2,0,c]$ the divergent part $q^4 R^2/\epsilon$ in each $\mathcal{J}_2$ cancels away to give a one-loop finite $m^2_{\phi_H}(0) \sim 1/R^2$. This cancellation is ensured by the equal number of bosonic and fermionic degrees of freedom, enforced by the initial supersymmetry. Using the leading behaviour of the functions $\mathcal{J}_{1,2}$ we find

$$m^2_{\phi_H}(q^2) = m^2_{\phi_H}(0) - \frac{(2f_{4,t})^2}{2^8} N_c\,(q^4R^2)\left[\frac{1}{\epsilon} + \ln(2\pi R\mu)\right] + \frac{1}{R^2}\mathcal{O}(q^2R^2) \tag{2.16}$$

where the μ-dependent term in the square bracket shows the regularisation scheme dependence induced by κ_ϵ. The finite part $\mathcal{O}(q^2R^2)$ can be evaluated from the second line in (2.15) using eqs. (A.2), (A.3) in the appendix to give the full running of the mass wrt momentum scale q^2.

Eq. (2.16) shows the presence in the sum of bosonic and fermionic contributions, of a pole multiplied by quartic dependence on (external momentum) q, originating from the two $\mathcal{J}_1$'s. The result is that the one loop scalar mass is not finite and has a UV divergence similar to that cancelled by (initial) supersymmetry in the J_2 dependent part. Therefore one must add in the action a higher derivative counterterm[7] to $m^2_{\phi_H}$ (recalling that ϕ_H is

[7]This result is similar to that in [13] which had instead 5D N=1 supersymmetry broken to N=0 on $S_1/(Z_2 \times Z_2')$.

$[Q]$	$[U]$	$[H_u]$	$[\lambda]$	# (no. of loops)
3/2 (bulk)	3/2 (bulk)	1 (brane)	-1	$n = 1$
3/2	1	3/2	-1	$n = 1$
3/2	3/2	3/2	-3/2	$n = 1$
3/2	1	1	-1/2	$n = 2$
1	1	3/2	-1/2	$n = 3$

Table 1: This is an *estimate* of the number of loops n when higher derivative operators may be generated. The localised superpotential $\int dy\,\delta(y)\,d^2\theta\,\lambda\,QUH_u$ with Q, U, H_u as brane/bulk fields generates higher derivative counterterms to the scalar zero mode of H_u (if a bulk field) or scalar component of H_u (if a brane field). An example is $\int d^4\theta\,(\lambda^2)^n H_u \Box H_u$. The table is a dimensional estimate of the number of loops when this counterterm arises located at $y = 0$, in function of the nature (bulk/brane) of the fields.

a brane field)

$$\int d^4x d^2\theta d^2\overline{\theta} \lambda_t^2\, H_u^\dagger \Box H_u \sim f_{4,t}^2 \int d^4x R^2 \phi_H^\dagger \Box^2 \phi_H + \cdots \tag{2.17}$$

The presence of the UV divergence and of corresponding higher derivative counterterm shows that, although the initial theory was supersymmetric, its non-renormalisable character is nevertheless manifest through a counterterm generated by the large number (multiplicity) of Kaluza-Klein modes which contribute to m_{ϕ_H}.

In table 1 we provided a general dimensional analysis for when higher derivative counterterms can arise from a localised superpotential. The table shows the number of loops in perturbation theory at which such counterterms can be generated, by assuming that any of the fields in the superpotential can be bulk or brane fields. This information can be used in realistic orbifold models, to avoid such operators at low orders in perturbation theory.

2.1.2 One-loop gauge correction to a brane scalar mass

Let us now consider the one-loop gauge contribution to the mass m_{ϕ_H} of the brane scalar ϕ_H located at $y = 0$, assumed to be charged under a U(1) group. This is induced by the action (2.7) and in the following we use eqs. (2.8) to (2.10). In the dimensional regularisation with $d = 4 - \epsilon$, bosonic (gauge) and fermionic (gauginos) contributions to the scalar self-energy at nonzero external momentum q^2, are respectively

$$-im_{\phi_H}^2(q^2)\Big|_B = (-i) 4 g_4^2 \mu^{4-d} \sum_{n\in\mathbf{Z}} \int \frac{d^d p}{(2\pi)^d} \frac{p.(q+p)}{(p^2 + n^2/R^2)(q+p)^2} \tag{2.18}$$

and

$$-im_{\phi_H}^2(q^2)\Big|_F = i 4 g_4^2 \mu^{4-d} \sum_{n\in\mathbf{Z}} \int \frac{d^d p}{(2\pi)^d} \frac{p.(q+p)}{(p^2 + (n+\rho)^2/R^2)(q+p)^2}\,. \tag{2.19}$$

Then, we find the one-loop corrections as

$$m_{\phi_H}^2(q^2)\Big|_B = \frac{g_4^2 (\mu\pi R)^\epsilon}{4\pi^3 R^2} \int_0^1 dx \left[\left(2 - \frac{\epsilon}{2}\right)\mathcal{G}_2[0,c] - \pi x(1-x)\, q^2 R^2\, \mathcal{G}_1[0,c]\right] \tag{2.20}$$

and

$$m^2_{\phi_H}(q^2)\Big|_F = -\frac{g_4^2(\mu\pi R)^\epsilon}{4\pi^3 R^2}\int_0^1 dx\left[\left(2-\frac{\epsilon}{2}\right)\mathcal{G}_2[\rho,c] - \pi x(1-x)\,q^2 R^2\,\mathcal{G}_1[\rho,c]\right] \quad (2.21)$$

with $c \equiv x(1-x)q^2R^2$ and we introduced

$$\mathcal{G}_j[\rho,c] \equiv \sum_{n\in\mathbf{Z}}\int_0^\infty \frac{dt}{t^{j-\epsilon/2}}e^{-\pi t[c+\beta(n+\rho)^2]}\,, \qquad j=1,2; \quad \beta = x\,. \quad (2.22)$$

After some calculations one obtains [30]

$$\begin{aligned}\mathcal{G}_1[\rho,c] &= -\ln\left|2\sin\pi(\rho+i\sqrt{c/\beta}\,)\right|^2, & (2.23)\\ \mathcal{G}_2[\rho,c] &= \frac{4\pi^2\,c^{3/2}}{3\,\beta^{1/2}} + \left\{(\beta\,c)^{1/2}\,\mathrm{Li}_2\left(e^{2i\pi(\rho+i\sqrt{c/\beta})}\right) + \frac{\beta}{2\pi}\,\mathrm{Li}_3\left(e^{2i\pi(\rho+i\sqrt{c/\beta})}\right) + \text{c.c.}\right\}.\end{aligned}$$

where $c,\,\beta>0$ and $\mathrm{Li}_\sigma(x)=\sum_{k\geq 1}x^k/k^\sigma$. Although these expressions are finite (no poles in ϵ), whenever one removes a mode from the series, poles arise and this justifies keeping the ϵ dependence explicit in their definition eq. (2.22).

Therefore, the resulting one-loop correction for the brane scalar is given by

$$\begin{aligned} m^2_{\phi_H}(q^2) = m^2_{\phi_H}(q^2)\Big|_B + m^2_{\phi_H}(q^2)\Big|_F &= \frac{g_4^2}{2\pi^3R^2}\int_0^1 dx\Big[\mathcal{G}_2[0,c]-\mathcal{G}_2[\rho,c]\Big] - \\ &- \frac{g_4^2}{4\pi^2R^2}(q^2\,R^2)\int_0^1 dx\,x(1-x)\Big[\mathcal{G}_1[0,c]-\mathcal{G}_1[\rho,c]\Big]. \end{aligned} \quad (2.24)$$

This result can be further simplified, but for our purpose it is enough to notice that it is finite (has no poles in ϵ). Therefore we conclude that no higher derivative counterterms are generated at the one-loop level. We note that the momentum-independent mass correction is given by

$$m^2_{\phi_H}(0) = \frac{g_4^2}{4\pi^4R^2}\left[\zeta[3] - \frac{1}{2}\Big(\mathrm{Li}_3(e^{2i\pi\rho}) + c.c.\Big)\right]. \quad (2.25)$$

This result agrees with the one obtained by changing the infinite Kaluza-Klein sum into a contour integral [17].

Alternatively, one can investigate the gauge correction by considering supergraphs. Given the presence of subtle differences between component and supergraph formalisms[8] we now use the supergraph formalism to show again that no higher derivative counterterms are present. This will confirm our conclusion obtained in the component formalism.

For this purpose we compute the gauge correction to the propagator of a (massless) brane chiral multiplet H in the absence of supersymmetry breaking. To do so we need to consider only one supergraph with brane-chiral and bulk-vector multiplets "running" in the loop [18]. We assume that, as in the 4D case, the soft breaking does not renormalise the

[8] related to gauge fixing in the WZ gauge in component formalism as compared to superfield gauge fixing in 5D.

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propagator of a massless brane chiral multiplet. With an appropriate gauge fixing term [19], i.e. the 5d version of the super Feynman gauge, the action for the vector superfield is

$$\mathcal{L}_5 = \int d^5x d^2\theta d^2\bar{\theta} V[-\Box - \partial_5^2]V\,. \tag{2.26}$$

Thus, the propagator for the bulk vector multiplet on S^1/Z_2 satisfies

$$(-q^2 + \partial_5^2)\Delta_5(y - y', \theta - \theta') = -\frac{1}{2}\sum_{n\in\mathbf{Z}} \delta(y - y' - 2\pi nR)\delta^4(\theta - \theta')\,. \tag{2.27}$$

Therefore, for $0 \le y - y' \le \pi R$, the mixed position-momentum propagator is given by

$$\Delta_5 = -\frac{1}{2}G_5(y, y')\delta^4(\theta - \theta') \qquad \text{with} \qquad G_5(y, y') = \frac{\cosh[q(y - y' - \pi R)]}{2\,q\sinh(\pi R q)}\,. \tag{2.28}$$

In particular, the bosonic part of the propagator at the origin with $y = y' = 0$ is given by

$$G_5(0,0) \;=\; \frac{1}{2q\tanh(\pi R\, q)} = \frac{1}{2\pi R}\sum_{n\in\mathbf{Z}} \frac{1}{q^2 + (n/R)^2}\,. \tag{2.29}$$

We obtain the one-loop gauge correction to the propagator of the brane chiral multiplet located at $y = 0$ as

$$-\frac{g_5^2}{2\pi R}\int \frac{d^4q}{(2\pi)^4} A(q)\left[\int d^4\theta \overline{H}(-q,\theta)H(q,\theta)\right] \tag{2.30}$$

where

$$\begin{aligned} A(q) \;&=\; \mu^{4-d}\sum_{n\in\mathbf{Z}}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(q+k)^2(k^2 + n^2/R^2)} \\ &=\; \frac{1}{(4\pi)^{d/2}(\pi\mu^2R^2)^{d/2-2}}\int_0^1 dx \sum_{n\in\mathbf{Z}}\int_0^\infty \frac{dt}{t^{d/2-1}} e^{-\pi t(c + xn^2)} \end{aligned} \tag{2.31}$$

where $c = x(1-x)\,q^2R^2$. Finally, with $d = 4 - \epsilon$ and using eqs. (2.22), (2.23), one finds that the one-loop gauge correction to the kinetic term of the brane scalar ϕ_H (component of H) in (2.30) has a momentum dependence of type

$$q^2\,A(q) = -\frac{1}{(4\pi)^2}\frac{1}{(\pi R)^2}\left[\frac{4y^3}{3} - \zeta[3] + 2y\text{Li}_2(e^{-2y}) + \text{Li}_3(e^{-2y})\right] + \mathcal{O}(\epsilon), \quad y = \pi R\sqrt{q^2}\,. \tag{2.32}$$

Since the above result has no poles in ϵ, we find, using this time the supergraph computation, that the wave function of the brane chiral multiplet is not renormalised. Therefore no higher derivative counterterms arise at one-loop order, in agreement with the previous computation using the component field formalism.

Finally, let us discuss the possible higher dimensional (derivative) operators which can be induced by the gauge corrections at higher loops, along the lines discussed in table 1. The corresponding operator for a brane chiral multiplet is

$$\int d^4\theta (g^2)^n \bar{H}\Box H$$

with n being the number of loops. Then, given the (mass) dimension of the higher dimensional gauge coupling, i.e. $[g_5^2] = -1$ in 5D higher dimensional operators can in principle be generated for $n = 2$ (two-loop). However, for a 6D case, $[g_6^2] = -2$, higher derivative operators may in principle be generated for $n = 1$ (one-loop).[9]

2.2 Higher derivative operators from non-local supersymmetry breaking

So far we have considered a brane-localised supersymmetry breaking, eqs. (2.3), (2.4), (2.7). In the following we consider a non-local supersymmetry breaking mechanism, with: (1) discrete and (2) continuous twisted boundary conditions, which we examine separately.

2.2.1 Discrete Scherk-Schwarz twists

First, let us impose that the 5D fields acquire under a $2\pi R$ shift, a phase which is the R-parity charge of these fields. The action of the R-parity operator is

$$\begin{aligned} Z_{2,R}Q(x,y,\theta) &= -Q(x,y,-\theta)\,, & Z_{2,R}Q^c(x,y,\theta) &= -Q^c(x,y,-\theta)\,,\\ Z_{2,R}H(x,y,\theta) &= H(x,y,-\theta)\,, & Z_{2,R}H^c(x,y,\theta) &= H^c(x,y,-\theta)\,,\\ Z_{2,R}V(x,y,\theta) &= V(x,y,-\theta)\,, & Z_{2,R}\Sigma(x,y,\theta) &= \Sigma(x,y,-\theta)\,. \end{aligned} \tag{2.33}$$

One also has a condition for U, U^c superfields similar to that for Q, Q^c. The condition for H, H^c stands for both $H_{u,d}, H^c_{u,d}$ and applies only if these fields are bulk fields. Eqs. (2.33), (2.1) give the spectrum relevant for our purpose

$$\begin{aligned} m_{\psi_M,k} &= \frac{k}{R}\,, \qquad k\geq 0\,, \qquad & m_{\psi^c_M,k} &= \frac{k}{R}, \qquad k\geq 1\\ m_{\phi_M,k} &= \frac{k+1/2}{R}\,, \qquad k\geq 0\,, \qquad & m_{\phi^c_M,k} &= \frac{k+1/2}{R}\,, \qquad k\geq 0\,; \qquad M = Q, U\,. \end{aligned} \tag{2.34}$$

and $\eta_k^{F_M} = \eta_k^{\phi_M} = 1$ and $\eta_k^{\psi_M} = 1/\sqrt{2}^{\delta_{k,0}}$. Also if the Higgs field is a bulk field, $m_{\phi_{H,k}} = k/R$ ($k \geq 0$) and $m_{\phi_{H^c,k}} = k/R$ ($k \geq 1$). Finally $m_{A_\mu,k} = k/R$, $m_{\lambda_{1,2,k}} = (k + 1/2)/R$ ($k \geq 0$).

Note that the spectrum on the orbifold S_1/Z_2 with the R-parity (2.33) has similarities with that in the case of $S_1/(Z_2 \times Z_2')$ orbifold (see for example [4]) with the Z_2' identified with a $Z_{2,R}$ Scherk-Schwarz breaking of supersymmetry. As a result the one-loop corrected $m_{\phi_H}(q^2)$ is expected to be similar to that in $S_1/(Z_2 \times Z_2')$ studied in[10] [13]. This similarity is only present when considering interactions from one fixed point, and breaks down when overlapping interactions from different fixed points are included, as can happen at higher loops.

The action is similar to that in eq. (2.11) with the remark that the first sum over k starts from $k = 0$. With this information we can compute the one-loop corrections to the mass of the scalar component ϕ_H of H_u, or of its zero mode if H_u is a bulk field. One has

[9] A component field computation on $T^2/\mathbf{Z}_2$ shows that one-loop gauge correction to the self-energy of a brane scalar is finite [20]. The agreement of this result with that using a supergraph approach is studied elsewhere [16].

[10] For the completeness of our analysis we include this case here in detail.

in this case a result similar to eq. (2.12), but note that the wavefunction coefficients η^{F_M} and the spectrum ($m_{\phi_Q^c}$) have changed. In euclidean space

$$-\left.im_{\phi_H}^2(q^2)\right|_B = -if_t^2 N_c \sum_{k\geq 0,\, l\geq 0}\left[\eta_k^{F_Q}\eta_l^{\phi_U}\right]^2\int\frac{d^dp}{(2\pi)^4}\frac{2\,(p+q)^2\mu^{4-d}}{((p+q)^2+m_{\phi_Q^c,k}^2)(p^2+m_{\phi_U,l}^2)}\,,$$
$$-\left.im_{\phi_H}^2(q^2)\right|_F = if_t^2 N_c \sum_{k\geq 0, l\geq 0}\left[\eta_k^{\psi_Q}\eta_l^{\psi_U}\right]^2\int\frac{d^dp}{(2\pi)^4}\frac{2\,p.(p+q)\;\mu^{4-d}}{((p+q)^2+m_{\psi_Q,k}^2)(p^2+m_{\psi_U,l}^2)}\,,\tag{2.35}$$

with the notation $f_t\equiv 2^n f_{4,t}$, where $n=1$ ($n=3/2$) is H_u is a brane (bulk) field.[11] In the first equation we used that the masses of ϕ_Q, ϕ_Q^c, ϕ_U, ϕ_U^c are all equal, and this explains the presence of a factor 2 in the numerator of the integrand.

The calculation of eq. (2.35) with replacements (2.34), proceeds as in section 2.1.1 (see also [13]). After adding the bosonic and fermionic contributions, one obtains

$$-\,m_{\phi_H}^2(q^2) = \frac{f_t^2}{16\pi^3R^2}N_c\int_0^1 dx\Big[\mathcal{J}_2[0,0,c]-\mathcal{J}_2[1/2,1/2,c]\Big]+\tag{2.36}$$
$$+\frac{f_t^2}{32\pi^2}\kappa_\epsilon N_c q^2\int_0^1 dx\Big[x(x-1)\mathcal{J}_1[0,0,c]-(1-x)^2\mathcal{J}_1[1/2,1/2,c]\Big].$$

The terms involving $\mathcal{J}_j[1/2,1/2,c]$, $j=1,2$ account for the bosonic contribution, while $\mathcal{J}_j[0,0,c]$ account for the fermionic part. The functions $\mathcal{J}_{1,2}$ are given in eq. (2.14). Comparing (2.36) to (2.15) one notices a similar structure, but there is a difference in the arguments of $\mathcal{J}_{1,2}$ in the two equations. Since the pole structure of $\mathcal{J}_{1,2}[c_1,c_2,c]$ does not depend on the arguments c_1, c_2, the discussion of the UV divergences is not changed from that of eq. (2.15). Eqs. (2.36), (2.14) give again

$$m_{\phi_H}^2(q^2) = m_{\phi_H}^2(0)-\frac{f_t^2}{2^8}N_c q^4R^2\left[\frac{1}{\epsilon}+\ln(2\pi R\mu)\right]+\frac{1}{R^2}\,\mathcal{O}(q^2R^2)\tag{2.37}$$

where $\mathcal{O}(q^2R^2)$ terms are due to $\mathcal{J}_1$ functions and can be evaluated numerically using the full expression of J_1 given in the appendix. The above result looks similar to that in (2.16), but the exact expression of $m_{\phi_H}^2(q^2)$ is different due to $\mathcal{J}_1$'s of different arguments. The counterterm has then the structure (when H_u is a bulk field)

$$\int d^4x dy\int d^2\theta d^2\overline{\theta}\delta(y)\lambda_t^2 H_u^\dagger\Box H_u \sim f_t^2\int d^4x\,R^2\sum_{n,p\geq 0}\phi_{H,n}^\dagger\Box^2\phi_{H,p}$$
$$\sim f_t^2\int d^4xR^2\phi_{H,0}^\dagger\Box^2\phi_{H,0}+\cdots\tag{2.38}$$

with $[\lambda_t]=-3/2$. If H_u is a brane field instead ($[H_u]=1$ and $[\lambda_t]=-1$), the counterterm reads

$$\int d^4xd^2\theta d^2\overline{\theta}\lambda_t^2 H_u^\dagger\Box H_u\sim f_t^2\int d^4xR^2\phi_H^\dagger\Box^2\phi_H+\cdots\,.\tag{2.39}$$

[11] If H_u is a bulk field, the above correction refers to its zero mode scalar.

2.2.2 Continuous Scherk-Schwarz twists.

Instead of a discrete twist eq. (2.33), in this section we impose continuous twists on bulk fields by using the $\mathrm{SU}(2)_R$ global symmetry. The $\mathrm{SU}(2)_R$ action under $y \to y + 2\pi R$ is

$$\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix}(x, y+2\pi R) = e^{-2\pi i\omega\sigma_2}\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix}(x,y)\,, \tag{2.40}$$

$$A_N(x, y+2\pi R) = A_N(x,y)\,, \qquad N = \mu, 5\,. \tag{2.41}$$

$$\begin{pmatrix}\phi_M\\ \phi_M^{c\dagger}\end{pmatrix}(x, y+2\pi R) = e^{-2\pi i\omega\sigma_2}\begin{pmatrix}\phi_M\\ \phi_M^{c\dagger}\end{pmatrix}(x,y)\,, \tag{2.42}$$

$$\begin{pmatrix}\psi_M\\ \psi_M^{c\dagger}\end{pmatrix}(x, y+2\pi R) = \begin{pmatrix}\psi_M\\ \psi_M^{c\dagger}\end{pmatrix}(x,y)\,, \qquad M \equiv Q, U\,. \tag{2.43}$$

where (ϕ_M, ϕ_M^c) and (ψ_M, ψ_M^c) are bulk quark multiplets, $M = Q, U$. If we also allow the Higgs multiplet(s) to live in the bulk, its boundary condition is

$$\begin{pmatrix}\phi_H\\ \phi_H^{c\dagger}\end{pmatrix}(x, y+2\pi R) = e^{-2\pi i\omega\sigma_2}\begin{pmatrix}\phi_H\\ \phi_H^{c\dagger}\end{pmatrix}(x,y)\,, \tag{2.44}$$

$$\begin{pmatrix}\psi_H\\ \psi_H^{c\dagger}\end{pmatrix}(x, y+2\pi R) = -\begin{pmatrix}\psi_H\\ \psi_H^{c\dagger}\end{pmatrix}(x,y) \tag{2.45}$$

where we note that the higgsinos acquire only a phase of R-parity because it is a singlet under $\mathrm{SU}(2)_R$. (If there are two Higgs multiplets in the bulk, one can also use $\mathrm{SU}(2)_H$ flavor symmetry to impose boundary conditions [5]).

The squarks (also the Higgs scalars if they are bulk fields) with a continuous Scherk-Schwarz phase have the mode expansion given by

$$\begin{pmatrix}\phi\\ \phi^{c\dagger}\end{pmatrix}(x,y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{\infty} u_n(y)\,\varphi_n(x) \tag{2.46}$$

where

$$u_n = e^{-i\omega\sigma_2 y/R}\begin{pmatrix}\cos\left(\frac{ny}{R}\right)\\ \sin\left(\frac{ny}{R}\right)\end{pmatrix} \tag{2.47}$$

and $(\Box - M_n^2)\varphi_n(x) = 0$ with $M_n^2 = (n+\omega)^2/R^2$. Here we note that the orthogonality is defined for eigenstates of $\mathrm{SU}(2)_R$ doublet as

$$\frac{1}{2\pi R}\int_0^{2\pi R} dy (u_n(y))^\dagger u_m(y) = \delta_{nm}\,. \tag{2.48}$$

The spectrum of the bulk fields is

$$m_{\psi_M,k} = \frac{k}{R}\,, \qquad k \geq 0\,, \qquad m_{\psi_M^c,k} = \frac{k}{R}\,, \qquad k \geq 1\,, \tag{2.49}$$

$$m_{\phi_M,k} = m_{\phi^c_M,k} = \frac{k+\omega}{R}\,, \qquad k\in \mathbf{Z}\,, \qquad (M\equiv Q,U)\,. \tag{2.50}$$

$$m_{\psi_H,k} = m_{\psi^c_H,k} = \frac{k+1/2}{R}\,, \qquad k\geq 0\,, \tag{2.51}$$

$$m_{\phi_H,k} = m_{\phi^c_H,k} = \frac{k+\omega}{R}\,, \qquad k\in \mathbf{Z}\,, \tag{2.52}$$

and $\eta^F_k = \eta^\phi_k = 1/\sqrt{2}$ and $\eta^\psi_k = 1/\sqrt{2}^{\delta_{k,0}}$. Finally, for gauginos one has that $m_{\lambda_{1,2},k} = (k+\omega)/R$, $(k\in\mathbf{Z})$. We note that the zero mode of a bulk Higgs scalar acquires a tree-level mass of ω/R. In this case, the one-loop Yukawa correction must be larger than this tree-level mass for electroweak symmetry breaking [6]. However, if the Higgs multiplet is a brane field, such situation is avoided, and one can still assume that there is no tree-level Higgs mass.

The action is in this case similar to that in eq. (2.11) except that the sums over k and l should be taken over the whole set of integer numbers. Then the one-loop mass corrections are

$$-\,i m^2_{\phi_H}(q^2)\Big|_B = -i f_t^2 N_c \sum_{k\in\mathbf{Z},l\in\mathbf{Z}} \Big[\eta^{F_Q}_k \eta^{\phi_U}_l\Big]^2 \int \frac{d^dp}{(2\pi)^4} \frac{2\,(p+q)^2\mu^{4-d}}{((p+q)^2+m^2_{\phi^c_Q,k})(p^2+m^2_{\phi_U,l})}\,,$$

$$-i m^2_{\phi_H}(q^2)\Big|_F = i f_t^2 N_c \sum_{k\geq 0,l\geq 0} \Big[\eta^{\psi_Q}_k \eta^{\psi_U}_l\Big]^2 \int \frac{d^dp}{(2\pi)^4} \frac{2p.(p+q)\mu^{4-d}}{((p+q)^2+m^2_{\psi_Q,k})(p^2+m^2_{\psi_U,l})}\,. \tag{2.53}$$

Adding the bosonic and fermionic contributions, one obtains the one-loop mass correction as

$$-\,m^2_{\phi_H}(q^2) = \frac{f_t^2}{16\pi^3R^2} N_c \int_0^1 dx \Big[\mathcal{J}_2[0,0,c] - \mathcal{J}_2[\omega,\omega,c]\Big] + \\ + \frac{f_t^2}{32\,\pi^2}\kappa_\epsilon N_c q^2 \int_0^1 dx\Big[x(x-1)\mathcal{J}_1[0,0,c] - (1-x)^2\mathcal{J}_1[\omega,\omega,c]\Big]. \tag{2.54}$$

Therefore we find again

$$m^2_{\phi_H}(q^2) = m^2_{\phi_H}(0) - \frac{f_t^2}{2^8} N_c q^4\, R^2 \left[\frac{1}{\epsilon} + \ln(2\pi R\,\mu)\right] + \frac{1}{R^2}\mathcal{O}(q^2R^2) \tag{2.55}$$

where $\mathcal{O}(q^2R^2)$ terms are ϵ-independent, are due to the two $\mathcal{J}_1$ functions in (2.54) and can be evaluated numerically using eqs. (A.2), (A.3). Higher derivative counterterms are again required, and the same arguments as in eq. (2.38), (2.39) apply.

In this section we considered so far Yukawa corrections only. Regarding the gauge correction, in both cases of discrete and continuous Scherk-Schwarz twists, the spectrum and brane coupling of gaugino are the same as in the local supersymmetry breaking. Hence the resulting one-loop correction to a brane scalar mass is the same [17] as before, eqs. (2.24) and (2.25).

3. Further remarks on higher derivative counterterms

In this section we discuss further the origin of the higher derivative counterterms and their relation to the local and non-local character of supersymmetry breaking of sections 2.1, 2.2.

Let us recall first that the radiative correction to the scalar mass from the Kaluza-Klein modes has the general structure given by (3.1) below. This equation is just a generalisation of eqs. (2.12) (2.13), (2.35), (2.53) for m_{ϕ_H}, all recovered for particular values of wavefunction coefficients $\sigma_{1,2} = 1, 2$ and of "mass shifts" $c_{1,2}$. After a long calculation one obtains[12] $(d = 4 - \epsilon)$

$$\begin{aligned}
\mathcal{E}(q^2) \equiv & \sum_{k_1 \geq 0; k_2 \geq 0} \left[\frac{\sigma_1}{2}\right]^{\delta_{k_1,0}} \left[\frac{\sigma_2}{2}\right]^{\delta_{k_2,0}} \int \frac{d^d p}{(2\pi)^d} \frac{\alpha p^2 + \beta (p.q) + \gamma q^2 + \delta}{[(q+p)^2 + (k_2 + c_2)^2/R^2][p^2 + (k_1 + c_1)^2\, u^2/R^2]} \\
= & \frac{1}{(4\pi)^2} \frac{2}{\epsilon} \bigg\{ \Big(\delta + q^2(\gamma - \beta/2)\Big)\Big(c_1 - \frac{\sigma_1 - 1}{2}\Big)\Big(c_2 - \frac{\sigma_2 - 1}{2}\Big) - \\
& - \frac{\alpha u}{6R^2}\bigg[u c_1 \Big(c_2 - \frac{\sigma_2 - 1}{2}\Big)\Big(1 + 3c_1(1 - \sigma_1) + 2c_1^2\Big) + \\
& + \Big(c_1 \leftrightarrow c_2, \sigma_1 \leftrightarrow \sigma_2, u \to \frac{1}{u}\Big)\bigg] - \\
& - \bigg[\frac{\pi^2}{32u} \delta q^2 R^2 + \frac{\pi^2}{2^8 u}(\alpha - 4\beta + 8\gamma)\, q^4 R^2\bigg]\bigg\} + \mathcal{O}(\epsilon^0)\,. \qquad (3.1)
\end{aligned}$$

This result is computed in the DR scheme and as usual, ϵ is a regulator of both the integral *and* the double series in front of it; ϵ is thus a genuine 6D regulator rather than a 4D one.[13] One firstly performs the momentum integral to obtain a double series whose summand (function of k_1, k_2) has powers involving ϵ and is multiplied by Gamma functions of ϵ-dependent argument. The series are analytically continued and their leading contribution to $\mathcal{O}(\epsilon^0)$ is obtained. Finally one takes account of the Gamma functions and finds the above result.[14] Note that the sums in (3.1) are restricted to positive integers only, unlike the expressions of $\mathcal{J}_{1,2}$ used previously and which involve double sums over $\mathbf{Z}$. The motivation and the advantage of using eq. (3.1) is that (unlike the analysis using $J_{1,2}$) it will allow us to see explicitly how quadratic divergences cancel.

The divergences in (3.1) are: q^2/ϵ which account for wave function renormalisation, the terms $1/(R^2\epsilon)$ which account for quadratic divergences and finally, the terms $q^4 R^2/\epsilon$ which account for quartic divergences (higher derivative counterterms).

We can now apply the result (3.1) to the calculations in sections 2.1 and 2.2 to discuss the origin of the higher derivative operators and of other divergences present. For the case in section 2.1.1 for the contribution in the first line of eqs. (2.12), (2.13) one has

$$\begin{aligned}
&\text{Bosonic part:} \qquad \sigma_1 = 2, \sigma_2 = 1, \beta = 2, \gamma = 1, c_1 = 1/2, c_2 = 0\,. \\
&\text{Fermionic part:} \qquad \sigma_1 = 1, \sigma_2 = 1, \beta = 1, \gamma = 0, c_1 = 0, c_2 = 0\,. \qquad (3.2)
\end{aligned}$$

while for the case in section 2.2.1, eq. (2.35) one has

$$\begin{aligned}
&\text{Bosonic part:} \qquad \sigma_1 = 2, \sigma_2 = 2, \beta = 2, \gamma = 1, c_1 = 1/2, c_2 = 1/2\,. \\
&\text{Fermionic part:} \qquad \sigma_1 = 1, \sigma_2 = 1, \beta = 1, \gamma = 0, c_1 = 0, \quad c_2 = 0\,. \qquad (3.3)
\end{aligned}$$

[12]To derive eq. (3.1) one can use the method outlined in appendix C of ref. [13].

[13]If the series in eq. (3.1) were restricted to a finite number of modes then ϵ would act as a 4D regulator only.

[14]In a similar way one can also find the finite, $\mathcal{O}(\epsilon^0)$ terms of (3.1).

In both cases $\alpha = 1$, $u = 1$, $\delta = 0$.

Finally, for the case in section 2.2.2 with a continuous Scherk-Schwarz phase, one has the same expression for the fermionic part as in (3.3), but the bosonic part is given by

$$\frac{1}{4}\Big[\mathcal{E}\Big(q^2; c_1 = c_2 = \omega\Big) + \mathcal{E}\Big(q^2; c_1 = \omega, c_2 = 1-\omega\Big) + \\ + \mathcal{E}\Big(q^2; c_1 = 1-\omega, c_2 = \omega\Big) + \mathcal{E}\Big(q^2; c_1 = c_2 = 1-\omega\Big)\Big] \tag{3.4}$$

where $\sigma_1 = \sigma_2 = 2, \beta = 2, \gamma = 1$ and $\alpha = 1, u = 1, \delta = 0$ in each term. Each contribution $\mathcal{E}$ in (3.4) has quadratic divergences $1/(R^2\epsilon)$, as seen from eq. (3.1). However they cancel in the sum of the first two terms (of arguments $c_2 = \omega$ and $c_2 = 1-\omega$) and also in the sum of the last two terms. This is essentially due to summing over the whole set $\mathbf{Z}$ of modes in eq. (2.46). Given the above values of $c_{1,2}, \sigma_{1,2}$, for all cases considered in eq. (3.2), (3.3), (3.4), one concludes that the coefficient of the quadratic divergences $1/(R^2\,\epsilon)$ vanishes separately for the bosons and fermions. Note that the values of $c_i = 0, 1/2$ are special for in that case (with corresponding values of σ_i), $\mathcal{E}$ alone has no quadratic divergence.

For the origin of higher derivative operators it is important to notice that the coefficient of q^4R^2/ϵ term is independent of $c_{1,2}$ and also of $\sigma_{1,2}$. Note that $c_{1,2}$ which enter the mass formulae for the Kaluza-Klein states are in fact set by the boundary conditions for the hypermultiplets with respect to the compact dimension. Therefore, this coefficient is independent of the phase that hypermultiplet fields have with respect to this dimension and, to some extent,[15] on the way supersymmetry is broken. The coefficient of q^4R^2/ϵ depends only on α, β, γ, which in turn are controlled by the nature (fermionic/bosonic) of the component fields (via their propagator in momentum space). Technically, the term q^4R^2/ϵ is strictly the result of the presence of *two* sums in front of the integral (3.1), over terms with $k_i \neq 0$, $i = 1, 2$. (this explains the absence of such divergences and corresponding higher dimensional counterterms in the 4D theory[16] of *zero-modes,* $k_i = 0$). Therefore, divergent terms q^4R^2/ϵ can be avoided at one-loop provided that there is only one bulk propagator (see table 1 for details when this can happen).

With these considerations one concludes that the presence of q^4R^2/ϵ and thus of the higher derivative counterterms is related to the multiplicity of the modes. Such operators are then due to the non-renormalisability of the models - initial supersymmetry cannot protect against their emergence as counterterms, regardless of the way supersymmetry is broken. Our analysis also shows that such operators are most relevant in models with low compactification scales, when their effects are less suppressed, see eqs. (2.17), (2.39).

Finally, note the presence of a u dependence of the coefficient of q^4R^2/ϵ in (3.1), which is present in the case of brane-localised supersymmetry breaking (see section 2.1). This signals some dependence between this supersymmetry breaking mechanism and the coefficient of the higher derivative operators. No such dependence appears for the discrete or continuous Scherk-Schwarz mechanism.

[15] see discussion in the last paragraph of this section.

[16] This is unlike the case of $1/(R^2\epsilon)$ divergences to which all modes including $k_i = 0$ contribute.

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4. Phenomenological implications: living with ghosts?

The presence of higher derivative operators in the action of the scalar field ϕ_H has implications for phenomenology. Their investigation is however difficult since theories with higher derivative operators can bring in more fundamental problems, such as unitarity violation, non-locality effects, the presence of additional ghost fields, and for these reasons such theories were less popular in the past (for some studies of such theories see for example [21]–[27]). Therefore the phenomenological considerations below should be taken with due care.

In the presence of higher derivative operators, new fields (ghosts) are present. To see this, let us write the propagator of ϕ_H in the presence of the higher derivative operator found in eqs. (2.17), (2.38), (2.39): $\mathcal{L}_4 = -\xi\, R^2\, \phi_H \Box^2 \phi_H + \cdots$, where ξ is an arbitrary constant (assumed positive for the convergence of the partition function); if ϕ_H is a bulk field, we refer to its zero-mode only. The propagator of ϕ_H changes then into

$$\frac{1}{-\xi R^2\, p^4 + p^2 - m^2} = \frac{1}{(1-4\xi R^2\, m^2)^{\frac{1}{2}}} \left[\frac{1}{p^2 - m_-^2} - \frac{1}{p^2 - m_+^2}\right] \tag{4.1}$$

with

$$m_\pm^2 = \frac{1}{2\xi R^2}\left[1 \pm (1-4\xi R^2\, m^2)^{\frac{1}{2}}\right] \tag{4.2}$$

The second term in the rhs of (4.1) has the "wrong" sign, thus it signals the presence in the model of a ghost field of mass m_+. Here m^2 is the one-loop induced mass $m^2_{\phi_H}(0)$ of ϕ_H plus the tree level contribution (if any), and m_-^2 is its value corrected by the higher derivative operator, but ignoring loop corrections $\mathcal{O}(q^2 R^2)$ of eq. (1.2) at $q^2 = m_-^2$. In function of the supersymmetry breaking mechanism which controls the coefficients $c_{1,2}$ in (2.14), one may have $m^2_{\phi_H}(0) < 0$ and thus electroweak symmetry breaking (assuming no tree level mass is present). With $m^2 < 0$, $\xi > 0$ then $m_+^2 > 0$ and $m_-^2 < 0$, and the symmetry breaking may be maintained.

In the following we require that $m_+^2 > M_*^2$, i.e. the ghost mass m_+ is larger than the cutoff M_* of our 5D theory, which for an effective theory approach as ours seems a natural requirement. We then study its implications for m_-^2. For this purpose we use a dimensional analysis [28, 29] to obtain perturbativity constraints on M_*, eqs. (A.30), (A.31), by requiring the effective gauge and Yukawa couplings be less than unity. The result is

$$M_* < \frac{12\pi^2}{\delta}\frac{1}{g_4^2 R}\,, \qquad M_* < \frac{12\pi^2}{\delta}\left[\frac{\delta}{16\pi^2\, f_{4,t}^{2/(p-1)}}\right]^{\frac{p-1}{p}} \frac{1}{R}\,. \tag{4.3}$$

The first (second) bound is from gauge (Yukawa) interaction. Here $p = 2$ ($p = 3$) if the Higgs field is a brane (bulk) field; δ is a factor equal to N for SU(N) gauge group, taking account of number of degrees of freedom present in loops [28]. Further, one imposes the first condition in the equation below, solves it for ξ which is then used to evaluate the change in m_-^2, to find:

$$m_+^2 = \gamma M_{*,u}^2\,, \Rightarrow \xi = \frac{\gamma\nu^2 - \beta sgn\,(m^2)}{\gamma^2 \nu^4}\,, \qquad \Delta_- = \frac{1}{sgn\,(m^2)\gamma\nu^2/\beta - 1}\,. \tag{4.4}$$

Here $M_{*,u}$ is upper bound of M_* in (4.3), ν is the factor multiplying $1/R$ in $M_{*,u}$, $\beta = |m^2|R^2$, *sgn(x)* is $+1\,(-1)$ for $x>0$ $(x<0)$, and Δ_- is the variation of m_-^2 relative to m^2. One should then take $\gamma > 1$ but $\gamma \sim \mathcal{O}(1)$ may still satisfy $m_+^2 > M_*^2$. We consider now the Yukawa correction only and take[17] $\delta = 3$ for $\mathrm{SU}(3)_c$, $m^2 < 0$ and $f_{4,t} \simeq 1$ for which $\beta \sim 0.1$. This gives for $\gamma = 1$, $\xi = 0.03(0.12)$ and $\Delta_- = -0.3\%\,(-1.2\%)$ for $p = 2\,(3)$. Therefore the coefficient of the higher derivative counterterm must be very small and its correction to the scalar field mass is negligible. A value $\gamma = 1/4$ would give $\xi = 0.13(0.53)$, $\Delta_- = -1.3\%\,(-4.8\%)$ for $p = 2$ (3), thus Δ_- is mildly changed. Note that this discussion ignored the additional terms ($\mathcal{O}(q^2R^2)$) at $q^2 = m_-^2$. For the implications on the *physical* mass of the scalar field and its dependence on the parameter ξ one must actually evaluate the minimum of the one-loop potential computed in the presence of such operators, which is beyond the purpose of this work. Finally, if the condition $m_+^2 > M_*^2$ is not satisfied, the presence of the ghost pole in the effective theory with the cutoff scale M_* requires further theoretical investigation.[18]

5. Conclusions

In this work we addressed the role that higher derivative operators play in the study of radiative corrections to the mass of the (Higgs) scalar field in 5D N=1 supersymmetric models compactified on S_1/Z_2. This is an important issue because it addresses how physics associated with compact dimensions decouples at low energies $q^2 \ll 1/R^2$, how these operators control the running of the scalar field mass across $q^2 \sim 1/R^2$, and the UV behaviour of the mass under the scaling of the momenta to $q^2 \gg 1/R^2$. Our technical results can be applied to a large class of orbifold models which investigate such one-loop effects from the bulk fields.

The interactions considered were localised superpotentials and gauge interactions. Although the models are rather minimal, the multiplet structure and the interactions considered are generic in any realistic higher dimensional extensions of the SM or its supersymmetric versions. It was found that Yukawa interactions, unlike gauge corrections, can generate higher derivative counterterms to the scalar field mass even at the one-loop level.

The work examined closely the relationship of the higher derivative operators with the way supersymmetry breaking was transmitted to the visible sector. The supersymmetry breaking scenarios addressed were local supersymmetry breaking by the F-term of a gauge singlet field localised at a hidden brane (fixed point) and also non-local breaking via Scherk-Schwarz (discrete/continuous) boundary conditions. In the case of local breaking, this effect is transmitted via radiative corrections to the scalar field, associated with the Higgs field. Such corrections, due to the compact dimension, are induced by the bulk fields which feel the supersymmetry breaking at the hidden brane. The second case, of non-local supersymmetry breaking, considered supersymmetry breaking by discrete and continuous Scherk-Schwarz twists of the bulk fields by using the $\mathrm{SU}(2)_R$ symmetry of the multiplet content.

[17] These correspond to the model with discrete Scherk-Schwarz supersymmetry breaking discussed above.
[18] for a discussion see [25].

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Our analysis showed that in all these cases of supersymmetry breaking the emergence of higher derivative counterterms to the mass of the scalar field is very similar and has little or no dependence on the details of the breaking mechanism considered. This is the main result of the paper. At the technical level this means that the coefficient of these operators is independent of the boundary conditions for the bulk fields. As a result these operators seem to be a generic presence in orbifold compactifications and initial supersymmetry cannot protect against their presence, in some cases even at the one-loop level. This result is ultimately due to the non-renormalisable character of the theory and also raises questions on the power of initial 5D supersymmetry in maintaining a mild UV behaviour in the compactified theory.

The phenomenological implications of the presence of higher derivative operators in the action of the scalar field were briefly investigated. It was found that the requirement that the 5D effective theory be weakly coupled together with the ghost field mass be larger than the 5D effective cutoff lead to small corrections on the scalar field mass in the action. This can change dramatically when any of these constraints is relaxed. To evaluate the *physical* mass one is required to compute the one-loop corrected scalar potential *in the presence* in the action of higher derivative operators. Such calculation requires however a prior and more comprehensive study of the theories with higher derivative operators at the quantum level, which is beyond our purpose.

Our study is also important because other approaches to compactification like string theory currently shed little light on such issues. The reason for this is that in string theory one computes the scalar potential (vacuum energy), derived at zero external momenta. Its second derivative giving a scalar mass cannot then recover a momentum dependence ("running") of the latter. We are thus confined to study these higher derivative operators in the framework of field theory orbifolds. The situation is very similar to the case of one-loop corrections to the gauge couplings, where again the role of higher derivative operators cannot be discussed in (on-shell) string loop corrections, but can be evaluated consistently in the context of field theory orbifolds [10 – 12].

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A. Appendix

A.1 Series of Integrals in the DR scheme

The functions $\mathcal{J}_{1,2}$ used in eqs. (2.14), (2.15), (2.36), (2.54) are, up to a re-definition of ϵ,

$$\mathcal{J}_p[c_1, c_2, c] \equiv \sum_{n_1, n_2 \in \mathbf{Z}} \int_0^\infty \frac{dt}{t^{p+\epsilon}} e^{-\pi t [c + a_1 (n_1 + c_1)^2 + a_2 (n_2 + c_2)^2]}, \quad a_{1,2} > 0, c \geq 0; \quad p = 1, 2. \tag{A.1}$$

The expressions given below for $\mathcal{J}_p$, $p=1,2$ generalise results quoted in appendix B of [13] valid only for the case $c/a_{1,2}\leq 1$ and which are included here for completeness. Using the method in appendix B of [30] one has that, if $0\leq c/a_1<1$

$$\begin{aligned}\mathcal{J}_1[c_1,c_2,c] &= \frac{\pi c}{\sqrt{a_1 a_2}}\frac{1}{\epsilon}+\frac{\pi c}{\sqrt{a_1 a_2}}\ln\left[4\pi a_1 e^{\gamma+\psi(\Delta_{c_1})+\psi(-\Delta_{c_1})}\right]+\\ &\quad +2\pi u\Big[\frac{1}{6}+\Delta_{c_1}^2-\Big(\frac{c}{a_1}+\Delta_{c_1}^2\Big)^{\frac{1}{2}}\Big]-\sum_{n_1\in\mathbf{Z}}\ln\left|1-e^{-2\pi\gamma(n_1)}\right|^2+\\ &\quad +\sqrt{\pi}u\sum_{p\geq 1}^{\infty}\frac{\Gamma[p+1/2]}{(p+1)!}\left[\frac{-c}{a_1}\right]^{p+1}\Big(\zeta[2p+1,1+\Delta_{c_1}]+\zeta[2p+1,1-\Delta_{c_1}]\Big),\end{aligned}\tag{A.2}$$

with $u\equiv\sqrt{a_1/a_2}$, $\gamma=0.577216...$ If $c/a_1\geq 1$, one has the same pole in ϵ, but different finite part:

$$\begin{aligned}\mathcal{J}_1[c_1,c_2,c] &= \frac{\pi c}{\sqrt{a_1 a_2}}\frac{1}{\epsilon}+\frac{\pi c}{\sqrt{a_1 a_2}}\ln\left[4\pi c e^{\gamma-1}\right]-\sum_{n_1\in\mathbf{Z}}\ln\left|1-e^{-2\pi\,\gamma(n_1)}\right|^2+\\ &\quad +4\left[\frac{c}{a_2}\right]^{\frac{1}{2}}\sum_{\tilde{n}_1>0}\frac{1}{\tilde{n}_1}\cos(2\pi\tilde{n}_1\,c_1)K_1\Big(2\pi\tilde{n}_1(c/a_1)^{\frac{1}{2}}\Big),\end{aligned}\tag{A.3}$$

with the notation

$$\begin{aligned}\gamma(n_1) &= \frac{1}{\sqrt{a_2}}[z(n_1)]^{\frac{1}{2}}-ic_2\,,\\ z(n_1) &= c+a_1(n_1+c_1)^2\,.\end{aligned}\tag{A.4}$$

Here $\zeta[z,a]$ is the Hurwitz Zeta function, $\zeta[z,a]=\sum_{n\geq 0}(n+a)^{-z}$, $\mathrm{Re}\,z>1$, $a\neq 0,-1,-2,\ldots$, and $\psi(x)=d/dx\ln\Gamma[x]$. Eqs. (A.2), (A.3) depend on the fractional part of $c_{1,2}$ defined by $\Delta_{c_i}\equiv c_i-[c_i]$ with $0\leq\Delta_{c_i}<1$, $[c_i]\in\mathbf{Z}$. Finally, K_n is the modified Bessel function [31]

$$\int_0^\infty dx x^{\nu-1}e^{-bx^p-ax^{-p}}=\frac{2}{p}\left[\frac{a}{b}\right]^{\frac{\nu}{2p}}K_{\frac{\nu}{p}}(2\sqrt{ab})\,,\qquad \mathrm{Re}(b)\,,\mathrm{Re}(a)>0\tag{A.5}$$

with

$$K_1[x]=e^{-x}\sqrt{\frac{\pi}{2x}}\left[1+\frac{3}{8x}-\frac{15}{128x^2}+\mathcal{O}(1/x^3)\right]\tag{A.6}$$

which is strongly suppressed at large argument.

One also finds that, if $c\ll 1$

$$\begin{aligned}\mathcal{J}_1[c_1,c_2,c\ll 1] &= \frac{\pi c}{\sqrt{a_1 a_2}}\frac{1}{\epsilon}-\ln\left|\frac{\vartheta_1(c_2-iuc_1|iu)}{(c_2-iuc_1)\eta(iu)}e^{-\pi u c_1^2}\right|^2-\ln(c/a_2+|c_2-iuc_1|^2)\\ \mathcal{J}_1[1/2,1/2,c\ll 1] &= \frac{\pi c}{\sqrt{a_1 a_2}}\frac{1}{\epsilon}-\ln\left|\frac{\vartheta_1(1/2-iu/2|iu)}{\eta(iu)}e^{-\pi u/4}\right|^2\\ \mathcal{J}_1[1/2,0,c\ll 1] &= \frac{\pi c}{\sqrt{a_1 a_2}}\frac{1}{\epsilon}-\ln\left|\frac{\vartheta_1(-iu/2|iu)}{(u/2)\eta(iu)}e^{-\pi u/4}\right|^2-\ln[u/2]^2\\ \mathcal{J}_1[0,0,c\ll 1] &= \frac{\pi c}{\sqrt{a_1 a_2}}\frac{1}{\epsilon}-\ln\left[4\pi^2|\eta(iu)|^4 a_2^{-1}\right]-\ln c\,,\qquad u\equiv\sqrt{a_1/a_2}\end{aligned}\tag{A.7}$$

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used in the text. Above we used the Dedekind Eta function

$$\eta(\tau) \equiv e^{\pi i\tau/12} \prod_{n\geq 1} (1-e^{2i\pi\tau n}) \,,$$
$$\eta(-1/\tau) = \sqrt{-i\,\tau}\,\eta(\tau) \,, \qquad \eta(\tau+1) = e^{i\pi/12}\eta(\tau) \,, \tag{A.8}$$

and the Jacobi Theta function ϑ_1 [31]

$$\vartheta_1(z|\tau) \equiv 2q^{1/8}\sin(\pi z)\prod_{n\geq 1}(1-q^n)(1-q^n e^{2i\pi z})(1-q^n e^{-2i\pi z}) \,, \qquad q \equiv e^{2i\pi\tau}$$
$$= -i\sum_{n\in\mathbf{Z}}(-1)^n e^{i\pi\tau(n+1/2)^2} e^{(2n+1)i\pi z} \tag{A.9}$$

which has the properties

$$\begin{aligned}
\vartheta_1(\nu|\tau+1) &= e^{i\pi/4}\vartheta_1(\nu|\tau) \,, \\
\vartheta_1(\nu+1|\tau) &= -\vartheta_1(\nu|\tau), \\
\vartheta_1(\nu+\tau|\tau) &= -e^{-i\pi\tau-2i\pi\nu}\vartheta_1(\nu|\tau) \\
\vartheta_1(-\nu/\tau|-1/\tau) &= e^{i\pi/4}\tau^{1/2}\exp(i\pi\nu^2/\tau)\vartheta_1(\nu|\tau) \,.
\end{aligned} \tag{A.10}$$

In the following we provide the results for $\mathcal{J}_2[c_1,c_2,c]$ whose calculation is almost identical. If $0 \leq c/a_1 < 1$

$$\begin{aligned}
\mathcal{J}_2[c_1,c_2,c] =& -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\frac{1}{\epsilon} - \frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\ln\left[4\pi a_1 e^{\gamma+\psi(\Delta_{c_1})+\psi(-\Delta_{c_1})}\right] + \\
&+\pi^2 a_2 u^3 \frac{1}{3}\left\{\frac{1}{15} - 2\Delta_{c_1}^2(1+\Delta_{c_1}^2) - 6\frac{c}{a_1}\left[\frac{1}{6}+\Delta_{c_1}^2\right] + 4\Big[c/a_1+\Delta_{c_1}^2\Big]^{\frac{3}{2}}\right\} + \\
&+\sum_{n_1\in\mathbf{Z}}\left\{(a_2 z(n_1))^{\frac{1}{2}}\mathrm{Li}_2(e^{-2\pi\gamma(n_1)}) + \frac{a_2}{2\pi}\mathrm{Li}_3(e^{-2\pi\gamma(n_1)}) + \text{c.c.}\right\} + \\
&+\pi^{3/2} a_2\, u^3 \sum_{p\geq 1}^{\infty}\frac{\Gamma[p+1/2]}{(p+2)!}\left[\frac{-c}{a_1}\right]^{p+2}\Big(\zeta[2p+1,1+\Delta_{c_1}] + \zeta[2p+1,1-\Delta_{c_1}]\Big).
\end{aligned} \tag{A.11}$$

If $c/a_1 \geq 1$ the pole structure is similar:

$$\begin{aligned}
\mathcal{J}_2[c_1,c_2,c] =& -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\frac{1}{\epsilon} - \frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\ln\left[\pi c e^{\gamma-3/2}\right] + \\
&+\sum_{n_1\in\mathbf{Z}}\left\{(a_2\, z(n_1))^{\frac{1}{2}}\mathrm{Li}_2(e^{-2\pi\gamma(n_1)}) + \frac{a_2}{2\pi}\mathrm{Li}_3(e^{-2\pi\gamma(n_1)}) + \text{c.c.}\right\} + \\
&+4cu\sum_{\tilde{n}_1>0}\frac{1}{\tilde{n}_1^2}\cos(2\pi\tilde{n}_1 c_1)\,K_2\Big(2\pi\tilde{n}_1(c/a_1)^{1/2}\Big).
\end{aligned} \tag{A.12}$$

For the simpler case $c \ll 1$,

$$\begin{aligned}
J_2[c_1,c_2,c\ll 1] =& -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\frac{1}{\epsilon} + \pi^2 a_2\, u^3\frac{1}{3}\left[\frac{1}{15} - 2\Delta_{c_1}^2(1-\Delta_{c_1})^2\right] + \qquad \text{(A.13)} \\
&+\sum_{n_1\in\mathbf{Z}}\left\{\sqrt{a_1 a_2}|n_1+c_1|\mathrm{Li}_2(e^{-2\pi\sigma_{n_1}}) + \frac{a_2}{2\pi}\mathrm{Li}_3(e^{-2\pi\sigma_{n_1}}) + \text{c.c.}\right\}
\end{aligned}$$

with $\sigma_{n_1} = i\, c_2 + \, u\, |n_1 + c_1|$. Also

$$J_2[0, 0, c \ll 1] \, = \, -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}} \frac{1}{\epsilon} + \frac{\pi^2}{45} a_2 u^3 + \\ + \frac{a_2}{\pi} \sum_{n_1 \in \mathbf{Z}} \Big\{ 2\pi u |n_1| \mathrm{Li}_2(e^{-2\pi\sigma_{n_1}}) + \mathrm{Li}_3(e^{-2\pi\sigma_{n_1}}) \Big\} \tag{A.14}$$

with $\sigma_{n_1} = u\, |n_1|$. Finally

$$J_2[1/2, 1/2, c \ll 1] \, = \, -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}} \frac{1}{\epsilon} - \frac{7\pi^2}{360} a_2 u^3 + \\ + \frac{a_2}{\pi} \sum_{n_1 \in \mathbf{Z}} \Big\{ 2\pi\, u\, |n_1 + 1/2| \mathrm{Li}_2(-e^{-2\pi\sigma_{n_1}}) + \mathrm{Li}_3(-e^{-2\pi\sigma_{n_1}}) \Big\} \tag{A.15}$$

with $\sigma_{n_1} = u\, |n_1 + 1/2|$.

A.2 Upper bounds on M_* from dimensional analysis

We derive the upper bounds on the cutoff M_*, used in the text, eqs. (4.3). For this purpose, we first outline the general dimensional analysis performed in [28] (section 3.2) and [29] (section 3). We then apply it to our case to derive upper bounds on M_* from perturbativity constraints.

The action in D dimensions (for orbifolds with y_i fixed points) has the form

$$\mathcal{L} = \mathcal{L}_{\mathrm{bulk}}(\phi) + \sum_i \delta^{D-4}(y - y_i) \mathcal{L}_i(\phi, \psi_i) \tag{A.16}$$

where ϕ (ψ) is a bulk (brane) field, respectively. One can assume canonical kinetic terms in (A.16) and then rescale these fields to their dimensionless counterparts $\hat{\phi}$, $\hat{\psi}$ and also the derivatives as

$$\phi(x, y) = \left(\frac{M_*^{D-2}}{l_D/\delta}\right)^{\frac{1}{2}} \hat{\phi}(x, y) \,, \qquad \psi_i(x) = \left(\frac{M_*^2}{l_4/\delta}\right)^{\frac{1}{2}} \hat{\psi}_i(x) \,, \qquad \partial = M_* \hat{\partial} \,, \tag{A.17}$$

where M_* is the cutoff scale; l_D is a suppression factor which accounts for angular integrations of loop corrections in D dimensions $l_D = (4\pi)^{D/2} \Gamma(D/2)$ which grows rapidly with D, and δ is the multiplicity of fields in loop diagrams for non-abelian groups, for example $\delta = N$ for SU(N), $\delta = 8$ for SO(10). Using eq. (A.17) in eq. (A.16) gives

$$\mathcal{L} = \frac{M_*^D}{l_D/\delta} \hat{\mathcal{L}}_{\mathrm{bulk}}(\hat{\phi}) + \sum_i \delta^{D-4}(y - y_i) \frac{M_*^4}{l_4/\delta} \hat{\mathcal{L}}_i(\hat{\phi}, \hat{\psi}_i) \,. \tag{A.18}$$

where $\hat{\mathcal{L}}_{\mathrm{bulk}}, \hat{\mathcal{L}}_i$ only contain dimensionless couplings and fields. If all couplings in $\hat{\mathcal{L}}_{\mathrm{bulk}}, \hat{\mathcal{L}}_i$ are of order 1, all loops are of the same order of magnitude. The theory with $\hat{\mathcal{L}}_{\mathrm{bulk}}, \hat{\mathcal{L}}_i$ remains weakly coupled if these dimensionless effective couplings remain smaller than unity [28, 29].

Let us apply this result to (bulk) gauge interactions. The relation between the D dimensional gauge coupling and the 4D gauge coupling is

$$\frac{V_{D-4}}{g_D^2} = \frac{1}{g_4^2} \tag{A.19}$$

where V_{D-4} is the volume of extra dimensions. From the rescaled covariant derivative,

$$\hat{D}_\mu = \frac{\partial_\mu}{M_*} - \frac{ig_D A_\mu}{M_*} = \frac{\partial_\mu}{M_*} - ig_D \left(\frac{M_*^{D-4}}{l_D/\delta}\right)^{\frac{1}{2}} \hat{A}_\mu \,, \tag{A.20}$$

we identify the dimensionless parameter corresponding to the gauge coupling as

$$g_{\rm eff}^2 \equiv g_D^2 \frac{M_*^{D-4}}{l_D/\delta} < 1 \,. \tag{A.21}$$

The above condition for perturbativity imposes the bound on the cutoff as

$$M_* < \left(\frac{l_D/\delta}{g_4^2 V_{D-4}}\right)^{\frac{1}{D-4}} . \tag{A.22}$$

Let us now consider a brane-localized interaction

$$\mathcal{L}_F = \int d^2\theta \, W(\Phi, \Psi) + \text{h.c.} \tag{A.23}$$

where Ψ (Φ) is a brane (Z_2 even bulk) multiplet respectively. Under the rescaling

$$\Phi(x,y) = \left(\frac{M_*^{D-2}}{l_D/\delta}\right)^{\frac{1}{2}} \hat{\Phi}(x,y) \,, \quad \Psi(x) = \left(\frac{M_*^2}{l_4/\delta}\right)^{\frac{1}{2}} \hat{\Psi}(x) \,, \quad d^2\theta = M_* d^2\hat{\theta} \,, \quad \theta = \frac{\hat{\theta}}{\sqrt{M_*}} \,, \tag{A.24}$$

the brane action becomes

$$\mathcal{L}_F = \frac{M_*^4}{l_4/\delta} \int d^2\hat{\theta} \hat{W}(\hat{\Phi}, \hat{\Psi}) + \text{h.c.} \,. \tag{A.25}$$

In particular, for the superpotential for the Yukawa interaction

$$W = \lambda_t \, H_u \, Q \, U^c, \qquad \text{with} \qquad \lambda_t = V_{D-4}^{p/2} \, f_{4,t} \tag{A.26}$$

where p is the number of bulk fields present in the Yukawa interaction, and $f_{4,t}$ is the 4D coupling. Then, the redefined superpotential is given by

$$\hat{W} = \lambda_{\rm eff} \hat{H}_u \hat{Q} \hat{U}^c \tag{A.27}$$

where

$$\lambda_{\rm eff} \equiv \lambda_t \, M_* \frac{l_4/\delta}{M_*^4} \left(\frac{M_*^{D-2}}{l_D/\delta}\right)^{\frac{p}{2}} \left(\frac{M_*^2}{l_4/\delta}\right)^{\frac{3-p}{2}} < 1 \,. \tag{A.28}$$

This condition gives the bound on M_*

$$M_* < \left[\frac{(l_D/\delta)^p (l_4/\delta)^{1-p}}{f_{4,t}^2 V_{D-4}^p}\right]^{\frac{1}{p(D-4)}} . \tag{A.29}$$

We now apply eqs. (A.22), (A.29) to our 5D models, to derive bounds on the 5D cutoff M_*. Eq. (A.22) gives

$$M_* < \frac{12\pi^2}{\delta g_4^2}\frac{1}{R}. \tag{A.30}$$

Eq. (A.29) becomes for the 5D case

$$M_* < \frac{12\pi^2}{\delta}\left[\frac{\delta}{16\pi^2 f_{4,t}^{2/(p-1)}}\right]^{\frac{p-1}{p}}\frac{1}{R} \tag{A.31}$$

where $p = 2, 3$. Eqs. (A.30), (A.31) were used in the text, eqs. (4.3).

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